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ON DISTRIBUTIVE LATTICES OF LEFT k-ARCHIMEDEAN SEMIRINGS

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Abstract. Here we introduce the notion of left k-Archimedean semirings which generalize the notion of k-Archimedean semirings [1], and characterize the semirings which are distributive lattices (chains) of left k-Archimedean semirings. A semiring S is a left k-Archimedean semiring if for all $a, b \in S,\ b \in \sqrt{Sa}$, the k-radical of Sa. A semiring S is a distributive lattice of left k-Archimedean semirings if and only if for all $a, b \in S,\ ab \in \sqrt{Sa}$ and S is a chain of left k-Archimedean semirings if and only if $\sqrt{L}$ is a completely prime k-ideal, for every left k-ideal L of S.

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Key words. k-radical, left k-Archimedean semiring, completely prime k-ideal, semiprimary k-ideal, congruence, distributive lattice congruence.

1. INTRODUCTION

In 1941, A. H. Clifford [4] first introduced and studied the semilattice decompositions of semigroups. The idea consists of decomposing a given semigroup S into component subsemigroups which are of simpler structure, through a congruence η on S such that the quotient semigroup S/η is the greatest semilattice homomorphic image of S and each η-class is a component subsemigroup. This well known result has since been generalized by M. S. Putcha, S. Bogdanović, M. Ćirić, F. Kmet and many others [3], [7], [8].

Both the greatest semilattice decomposition of semigroups and the greatest distributive lattice decomposition of semirings evolve out of the divisibility relation. In an additive idempotent semiring S, we define $a \longrightarrow b$ if $a \mid b^n$ for some $n \in \mathbb{N}$. The binary relation $\longrightarrow$ is neither symmetric nor transitive in general, which allows us to find the least distributive lattice congruence as the least congruence from $\longrightarrow$ in several ways. For example, symmetric opening of the transitive closure and the transitive closure of the symmetric opening of $\longrightarrow$ give us different description of the least distributive lattice congruence on S. Such variations in the description of the least distributive

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